HW 13 Problem 1

Zipline Problem Statement:

A zipline company, Skyline Eco-Adventures, sponsored a clinic team to design an active speed control system for riders of ziplines. In this problem, we will be attempting a variation of this problem, where the goal is to determine the optimal vertical drop and tension for the wire rope, such that with no wind, certain conditions are met. These conditions are that a 30 kg, 70 kg, and 140 kg rider will all make it to the end of the zipline and not have an exit velocity exceeding 5 m/s, all while maximizing the speed of the 70 kg rider to make the ride as fun as possible. After determining the catenary, the curved shape of a uniform rope hanging from two supports, use the provided zipline paper, as well as given differential equations, to determine the optical zipline tension and vertical drop between the end supports, the maximum velocity and exit velocity for 30 kg, 70 kg, and 140 kg riders, as well as create plots of each rider's position and velocity profile as a function of position on the zipline.

Pages 3-6 detail the steps and process I took to complete the steps. The final answer is on page 7, followed by a write up on page 8.

GOALS:

1. Determine optimal vertical drop between end supports AND zipline tension such that the velocity of the 70 kg rider is maximized.
2. Calculate maximum velocity and exit velocity for all three riders.
3. Plot each rider's path AND velocity profile as a function of position on the zipline.

MORE INFO:

Decision variables:

1. zipline tension
2. Vertical drop between the end supports

Objective function:

- peak velocity of the 70 kg rider
  → steeper the slope at the beginning, the faster the rider will go

Constraints:

1. exit velocity of all three riders must be less than 5 m/s
2. all three riders must make it to the end of the zipline (not get stuck)
3. wire rope tension must be less than 50 kN
4. 10 m ≤ vertical drop ≤ 40 m

} use fmincon upper & lower bounds

PROPOSED STEPS:

1. At different f-values ranging from 0 to 1, use fsolve to find a and b.
2. At each f, solve for $\tilde{x}$ and $\tilde{y}$ using the value of a and b at that f.
3. Convert the $\tilde{x}$ and $\tilde{y}$ values to x and y values.
4. Now that we have a dataset of rider position values, use spline and ppval commands to interpolate the data.
5. Calculate the derivative of the data in step 4, aka $\frac{dy}{dx}$, using the fnder command
6. Combine the differential equations given in the problem statement to develop a singular differential equation to solve for dv^2/dx.
7. Solve the differential equation in step 6 to get a solution for v^2
8. Take the square root of v^2 found in step 7
9. Figure out which constraints to implement.
10. Plan out implementation of constraints in MATLAB using fmincon
11. Implement optimization in MATLAB
12. Use the calculated optimal tension and vertical drop to find the maximum and exit velocities for all riders
13. Plot each rider's path and velocity profile

STEP 1:

To find a and b at different values of f, we can use eqn 8 + eqn 16 = eqn 9 and eqn 10 + eqn 17 = eqn 11. This gives us two equations with two unknowns: a & b.

$$\tilde{x} + \tilde{x}' = \tilde{X}$$

equation 8, contains a, b, and f

equation 16, contains a, b, f, and γ

equation 9, contains a_0 and b_0

$$\tilde{y} + \tilde{y}' = \tilde{Y}$$

equation 10, contains a, b, and f

equation 17, contains a, b, f, and γ

equation 11, contains a_0 and b_0

$$\tilde{x} + \tilde{x}' - \tilde{X} = 0 \qquad \tilde{y} + \tilde{y}' - \tilde{Y} = 0$$

→ use already calculated a_0 and b_0 values, for the "no rider" case

→ $\gamma = \frac{M}{m} = \frac{\text{rider's mass}}{\text{cable's mass}}$ $\qquad m = 15 \frac{N}{m} \cdot L$ ← calculated length of rope

STEP 2:

Plug in each a and b value from step 1 into equations for $\tilde{x}$ and $\tilde{y}$ to get a set of $\tilde{x}$ and $\tilde{y}$ values at varying f values.

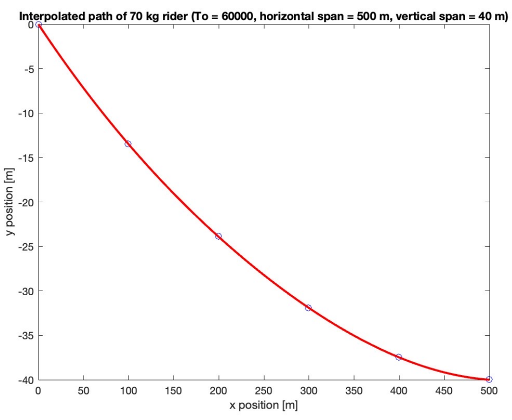
★ DO IT IN THE SAME FOR LOOP ★

STEP 3:

$x = \tilde{x} L$ and $y = \tilde{y} L$

STEP 4:

dataset interpolated in MATLAB, produces the following plot:



STEP 5:

$\frac{dy}{dx}$ evaluated in MATLAB

STEP 6:

$$m\frac{dv}{dt} = mg\sin(-\alpha) - F_{drag}$$

$$\frac{dv}{dt} = g\sin(-\alpha) - \frac{F_{drag}}{m}$$

plugging in the following equations in $\frac{dv}{dt}$:

$\alpha = \tan^{-1}\left(\frac{dy}{dx}\right)$ and $F_{drag} = \frac{1}{2}\rho C_d A V^2$

$$= \frac{1}{2}(1.2)(1)(0.05\, m^{2/3})\, v^2$$

$$= 0.03\, m^{2/3}\, v^2$$

$$\frac{dv}{dt} = g\sin\left(\tan^{-1}\left(\frac{dy}{dx}\right)\right) - \frac{0.03\, m^{2/3}\, v^2}{m}$$

$$\frac{dv}{dt} = g\sin\left(\tan^{-1}\left(\frac{dy}{dx}\right)\right) - \frac{0.03\, v^2}{m^{1/3}}$$

$$\frac{d(v^2)}{dx} = 2\frac{dv}{dt}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\frac{d(v^2)}{dx} = 2\left[g\sin\left(-\tan^{-1}\left(\frac{dy}{dx}\right)\right) - \frac{0.03\, v^2}{m^{1/3}}\right]\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

STEP 7 & STEP 8:

done in MATLAB

STEP 9:

constraints:

① all riders must make it to the end of the zipline

- if the 30 kg rider makes it to the end, then all of them will
- exit velocity² of the 30 kg rider must be greater than zero

$$V_{exit}^2(30) \leq 0$$

② exit velocity of all three riders must be less than 5 m/s

- heavier individuals will have a higher exit velocity. therefore, as long as the 140 kg rider has an exit velocity less than 5 m/s, they all will.
- exit velocity of the 140 kg rider must be less than 5 m/s

$$V_{exit}^2(140) \leq 25 \quad \text{so} \quad V_{exit}^2(140) - 25 \leq 0$$

③ wire rope tension must be less than 50 kN

④ $10\text{ m} \leq$ vertical drop $\leq 40\text{ m}$

STEP 10

★ using the nonlcon version of fmincon

- "fun" is the objective function outputting the maximum exit velocity of 70 kg (but making it negative to maximize it)
  - objective function should contain/depend on two variables, vertical drop and tension, because lower and upper bounds need to be placed on them

- "xØ" = [20, 40,000] → initial guesses for vertical drop and tension (within bounds)

- A, b, Aeq, beq are all [ ] b/c there are no linear constraints

- lb are lower bounds of drop & tension $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$ → lower bound of drop → lower bound of tension, chosen to be zero b/c we don't want negative tension

- ub are upper bounds of drop & tension $\begin{bmatrix} 40 \\ 50{,}000 \end{bmatrix}$ → upper bound of drop → upper bound of tension

- nonlcon contains the nonlinear inequalities c(x) or equalities ceq(x) that the minimization is subject to
  - ★ fmincon will optimize such that $c(x) \le 0$ and $ceq = 0$. for us:

(-) b/c V_{exit}^2 has to actually be greater than zero

$-V_{exit}^2(30) \le 0$

$c(x)_1 = -V_{exit}^2(30)$

$V_{exit}^2(140) - 25 \le 0$

$c(x)_2 = V_{exit}^2(140) - 25$

ceq is empty b/c there are no nonlinear equality constraints

STEP 11, 12, 13

done in MATLAB

ANSWERS

optimal tension: 39,571 N

optimal vertical drop: 40 m

for 30 kg rider:

maximum velocity: 11.5308 m/s

exit velocity: 0.0272 m/s

for 70 kg rider:

maximum velocity: 12.8849 m/s

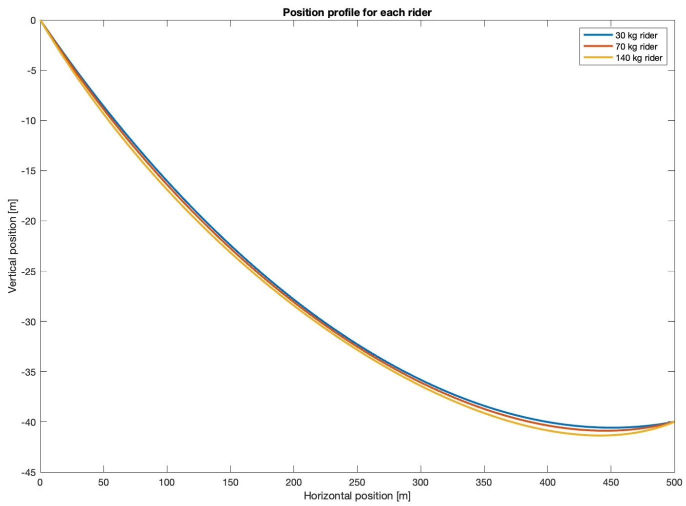
exit velocity: 2.1833 m/s

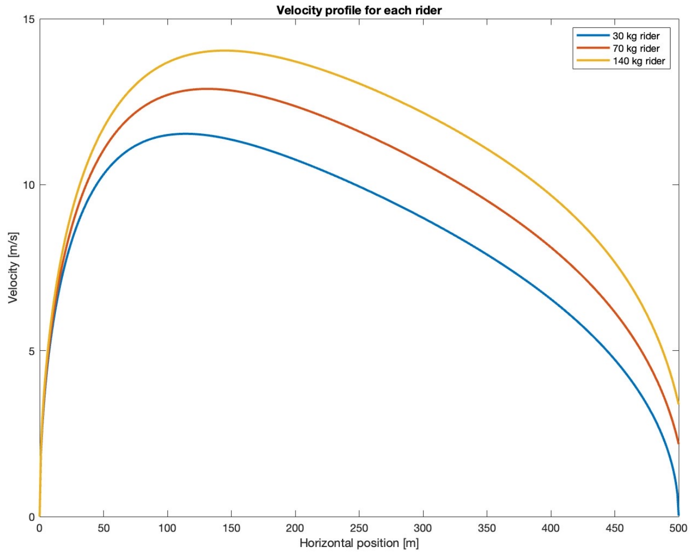
for 140 kg rider:

maximum velocity: 14.0375 m/s

exit velocity: 3.3781 m/s

GRAPHS:





· WRITE - UP ·

First, the decision variables, objective function, and constraints were all determined for this problem. There were two decision variables in this problem were the tension of the zipline and the vertical drop between the end supports. The objective function, which is what I was trying to optimize, was the peak velocity of the 70 kg rider. The constraints were that the exit velocity of all three riders must be less than 5 m/s, all three riders must make it to the end of the zipline, the wire rope tension must be less than 50 kN, and the vertical drop must be between 10 and 40 m. The goal of the problem was to determine the optimal vertical drop between the end supports and the zipline tension such that all the aforementioned conditions were met.

After using the equations in the zipline paper, as well as the differential equations provided in the problem statement, I found the following things: the catenary shape of the zipline with no riders, the position of riders (of different weights) on the zipline, and the velocity of riders (of different weights) on the zipline. More information on the steps I took to find these are detailed above.

I decided to use MATLAB's fmincon command to implement the constraints. I used the nonlcon version of fmincon because this problem had nonlinear constraints, which are implemented through nonlcon. The first constraint I looked at was that all riders must make it to the end of the zipline. Using simple physics, I reasoned that if the lightest rider (30 kg in our case) makes it to the end of the zipline, then all of them will make it to the end as well. This is because heavier riders have a higher exit speed, while lighter riders are more likely to get stuck somewhere on the zipline. If a rider doesn't make it to the end of the zipline, the square of their exit velocity will be negative. Therefore, the constraint I implemented was that the square of the exit velocity of the 30 kg rider must be greater than zero, in which case all of the riders will make it to the end of the zipline. The second constraint I implemented was that the exit velocity of all three riders must be less than 5 m/s. Using the same physics reasoning, since the heaviest rider (140 kg in our case) will have the largest exit velocity, as long as their exit velocity is less than 5 m/s, all of the riders will pass this constraint. Therefore, the constraint I implemented was that the square of the velocity of the 140 kg rider must be less than 25 m/s. Since nonlcon contains the nonlinear inequalities and equalities that the minimization is subject to, the two constraints I described in this paragraph were implemented through nonlcon. The other two constraints are that the wire rope tension must be less than 50 kN and the vertical drop must be between 10 and 40 m. Since these are constraints on the decision variables, I used the lower and upper bounds on fmincon to implement these. I set the vertical drop's lower bound to be 10 m and its upper bound to be 40 m. I set the tension's upper bound to be 50,000 and its lower bound to be 0, because I thought it would not be appropriate for the tension to be negative.

The answers I got at the end of this problem are specified on the page above. They satisfy all of the constraints, because all of the exit velocities are under 5 m/s, the tension of the rope is under 50 kN, and the optimal vertical drop is between 10 and 40 m, inclusive. I also know that I optimized the velocity of the 70 kg rider. I know that fmincon tries to find the minimum (or in our case, the maximum) based on the starting point we provide. Sometimes, it might think it found the minimum point, but it only found a local minimum, and not the global minimum. In order to ensure my solution found the global (and not just a local) minimum, I provided different initial guesses for the decision variables (vertical drop and tension). I found the velocity at the optimized tension and vertical drop with these guesses. After multiple guesses, I realized that the velocity I got was in fact the global maximum, because I did not find a higher velocity. If I had more time, to streamline this process, I would create a 3D plot of the velocities for the 70 kg rider to ensure the maximum that I found was in fact the global maximum.